Paper Reference(s)

## 6664/01

## Edexcel GCE

## Core Mathematics C2

## Bronze Level B3

## Time: 1 hour 30 minutes

$\frac{\text { Materials required for examination }}{\text { Mathematical Formulae (Green) }} \quad \frac{\text { Items included with question papers }}{\mathrm{Nil}}$

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 73 | 67 | 61 | 55 | 48 | 42 |

1. A geometric series has first term $a=360$ and common ratio $r=\frac{7}{8}$.

Giving your answers to 3 significant figures where appropriate, find
(a) the 20th term of the series,
(b) the sum of the first 20 terms of the series,
(c) the sum to infinity of the series.

January 2012
2.

$$
y=\frac{x}{\sqrt{ }(1+x)}
$$

(a) Complete the table below with the value of $y$ corresponding to $x=1.3$, giving your answer to 4 decimal places.
(1)

| $x$ | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0.7071 | 0.7591 | 0.8090 |  | 0.9037 | 0.9487 |

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an approximate value for

$$
\int_{1}^{1.5} \frac{x}{\sqrt{ }(1+x)} \mathrm{d} x
$$

giving your answer to 3 decimal places.
You must show clearly each stage of your working.
3. Find the first 4 terms, in ascending powers of $x$, of the binomial expansion of

$$
\left(2-\frac{1}{2} x\right)^{8}
$$

giving each term in its simplest form.
4.

$$
\mathrm{f}(x)=x^{3}+a x^{2}+b x+3 \text {, where } a \text { and } b \text { are constants. }
$$

Given that when $\mathrm{f}(x)$ is divided by $(x+2)$ the remainder is 7 ,
(a) show that $2 a-b=6$.

Given also that when $\mathrm{f}(x)$ is divided by $(x-1)$ the remainder is 4 ,
(b) find the value of $a$ and the value of $b$.
5.

## Figure 1



Figure 1 shows 3 yachts $A, B$ and $C$ which are assumed to be in the same horizontal plane. Yacht $B$ is 500 m due north of yacht $A$ and yacht $C$ is 700 m from $A$. The bearing of $C$ from $A$ is $015^{\circ}$.
(a) Calculate the distance between yacht $B$ and yacht $C$, in metres to 3 significant figures.

The bearing of yacht $C$ from yacht $B$ is $\theta^{\circ}$, as shown in Figure 1.
(b) Calculate the value of $\theta$.

January 2008
6.


Figure 1
The line with equation $y=10$ cuts the curve with equation $y=x^{2}+2 x+2$ at the points $A$ and $B$ as shown in Figure 1. The figure is not drawn to scale.
(a) Find by calculation the $x$-coordinate of $A$ and the $x$-coordinate of $B$.

The shaded region $R$ is bounded by the line with equation $y=10$ and the curve as shown in Figure 1.
(b) Use calculus to find the exact area of $R$.
7. (i) Find the exact value of $x$ for which

$$
\begin{equation*}
\log _{2}(2 x)=\log _{2}(5 x+4)-3 . \tag{4}
\end{equation*}
$$

(ii) Given that

$$
\log _{a} y+3 \log _{a} 2=5
$$

express $y$ in terms of $a$.
Give your answer in its simplest form.
8. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75 \pi$ $\mathrm{cm}^{3}$.

The cost of polishing the surface area of this glass cylinder is $£ 2$ per $\mathrm{cm}^{2}$ for the curved surface area and $£ 3$ per cm ${ }^{2}$ for the circular top and base areas.

Given that the radius of the cylinder is $r \mathrm{~cm}$,
(a) show that the cost of the polishing, $£ C$, is given by

$$
C=6 \pi r^{2}+\frac{300 \pi}{r} .
$$

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.
(c) Justify that the answer that you have obtained in part (b) is a minimum.

May 2015
9. The curve with equation

$$
y=x^{2}-32 \sqrt{ } x+20, \quad x>0
$$

has a stationary point $P$.
Use calculus
(a) to find the coordinates of $P$,
(b) to determine the nature of the stationary point $P$.
10.


Figure 2
The finite region $R$, as shown in Figure 2, is bounded by the $x$-axis and the curve with equation

$$
y=27-2 x-9 \sqrt{ } x-\frac{16}{x^{2}}, \quad x>0
$$

The curve crosses the $x$-axis at the points $(1,0)$ and $(4,0)$.
(a) Copy and complete the table below, by giving your values of $y$ to 3 decimal places.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| $y$ | 0 | 5.866 |  | 5.210 |  | 1.856 | 0 |

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of $R$, giving your answer to 2 decimal places.
(c) Use integration to find the exact value for the area of $R$.

## END




| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 7 (a) | $\begin{aligned} & \log _{2}\left(\frac{2 x}{5 x+4}\right)=-3 \text { or } \log _{2}\left(\frac{5 x+4}{2 x}\right)=3, \text { or } \log _{2}\left(\frac{5 x+4}{x}\right)=4 \\ & \left(\frac{2 x}{5 x+4}\right)=2^{-3} \text { or }\left(\frac{5 x+4}{2 x}\right)=2^{3} \text { or }\left(\frac{5 x+4}{x}\right)=2^{4} \text { or } \end{aligned}$ | M1 |
|  | $\left(\log _{2}\left(\frac{2 x}{5 x+4}\right)\right)=\log _{2}\left(\frac{1}{8}\right)$ <br> $16 x=5 x+4 \Rightarrow x=$ (depends on previous Ms and must be this equation or equivalent) $x=\frac{4}{11}$ or exact recurring decimal $0 . \dot{3} \dot{6}$ after correct work | dM1 <br> A1cso <br> (4) |
| (b) | $\begin{aligned} & \log _{a} y+\log _{a} 2^{3}=5 \\ & \log _{a} 8 y=5 \\ & y=\frac{1}{8} a^{5} \end{aligned}$ <br> Applies product law of logarithms. $y=\frac{1}{8} a^{5}$ | M1 <br> dM1 <br> A1cao |
|  |  | (3) <br> [7] |


| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 8 (a) | Either: (Cost of polishing top and bottom (two circles) is ) $3 \times 2 \pi r^{2}$ or (Cost of polishing curved surface area is) $2 \times 2 \pi r h$ or both - just need to see at least one of these products Uses volume to give $(h=) \frac{75 \pi}{\pi r^{2}} \quad$ or $\quad(h=) \frac{75}{r^{2}}$ (simplified) (if $V$ is misread - see below) $\begin{array}{l\|l} (C)=6 \pi r^{2}+4 \pi r\left(\frac{75}{r^{2}}\right) & \begin{array}{l} \text { Substitutes expression for } h \text { into } \\ \text { area or cost expression of form } \\ A r^{2}+B r h \end{array} \\ C=6 \pi r^{2}+\frac{300 \pi}{r} & * \end{array}$ | B1 <br> B1ft <br> M1 <br> A1* |
| (b) | $\left\{\frac{\mathrm{d} C}{\mathrm{~d} r}=\right\} 12 \pi r-\frac{300 \pi}{r^{2}}$ or $12 \pi r-300 \pi r^{-2}$ (then isw) $12 \pi r-\frac{300 \pi}{r^{2}}=0$ so $r^{k}=$ value where $k= \pm 2, \pm 3, \pm 4$ <br> Use cube root to obtain $r=\left(\text { their } \frac{300}{12}\right)^{\frac{1}{3}}(=2.92)$ - allow $r=3$, and thus $C=$. <br> Then $C=$ awrt 483 or 484 | M1 A1 ft dM1 ddM1 Alcao |
| (c) | $\left\{\frac{\mathrm{d}^{2} C}{\mathrm{dr} r^{2}}=\right\} 12 \pi+\frac{600 \pi}{r^{3}}>0$ so minimum | B1ft <br> (1) <br> [10] |
| 9 (a) | $\left\{\frac{\mathrm{d} y}{\mathrm{~d} x}=\right\} 2 x-16 x^{-\frac{1}{2}}$ <br> $2 x-16 x^{-\frac{1}{2}}=0 \Rightarrow x^{\frac{3}{2}}=, x^{-\frac{3}{2}}=$,or $2 x-=16 x^{-\frac{1}{2}}$ then squared then obtain $x^{3}=$ <br> [or $2 x-16 x^{-\frac{1}{2}}=0 \Rightarrow x=4$ (no wrong work seen)] $\left(x^{\frac{3}{2}}=8 \Rightarrow\right) x=4$ <br> $x=4, y=4^{2}-32 \sqrt{4}+20=-28$ (ignore $y=100$ as second answer) | M1 A1 <br> M1 <br> A1 <br> M1 A1 <br> (6) |
| (b) | $\left\{\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=\right\} 2+8 x^{-\frac{3}{2}}$ <br> $\left(\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}>0 \Rightarrow\right) y$ is a minimum ( there should be no wrong reasoning) | M1 A1 <br> A1 <br> (3) |



## Examiner reports

## Question 1

This was a very straightforward first question on Geometric Progressions. Over $80 \%$ of candidates obtained full marks. Parts a) and c) were done exceptionally well with most of the problems arising in part $b$ ), where a sizeable group of candidates who had used the power ( $n-1$ ) or 19 in a) then used it again in b) instead of the correct $n=20$. Other loss of marks was usually as a result of calculator operation errors and rounding, some candidates offering 268 and 288 as answers to b) and c) respectively.

There were fewer candidates confusing geometric and arithmetic series formulae than in previous years, but the question did tell them what the series was. On the whole, the GCE series work seemed to be well applied by the majority but GCSE rounding caused more problems.

## Question 2

In part (a) almost all achieved the first mark for 0.8572 . The value 0.8571 was seen rarely, as was 0.857 . These answers did not get this mark.

In part (b) the main error was in calculating the strip width; $\frac{1}{12} 1 / 12$ coming from doing the calculation $(1.5-1) \div 6$.

The common bracketing error, with brackets omitted, appeared relatively frequently. This usually led to errors in the calculation. There were occasional, but rare, errors with extra values repeated in the innermost bracket, or 0 included as the first value. There were some who tried to integrate to produce an answer (but got nowhere) and others who split it up into several integrals to attempt to evaluate, again with little success.

Usually, however, this question was answered correctly.

## Question 3

This question was done well and the majority of candidates gained full marks. The method used was equally divided between candidates working with the expression given and those taking out a factor of 2 at the start. Although answers using the second approach were more likely to have errors, most candidates could work accurately with this method. The common error of applying the power of a bracket to the $x$ but not to the $-\frac{1}{2}$ was only seen occasionally. The most common error seen was where candidates used $\frac{1}{2} x$ instead of $-\frac{1}{2} x$. Some candidates gave every term of the expansion, and not just the ones required by the question, which would cost them time in an examination.

## Question 4

The vast majority of candidate used the remainder theorem correctly in this question and there were very few correct attempts at the alternative method of long division. $77 \%$ of candidates achieved full marks.

Part (a): Most candidates gained both marks for this part of the question. The main errors were with the minus signs and a few did not actually equate their expression to 7 .

Part (b): The majority of candidates again used the remainder theorem correctly and then solved the simultaneous equations to obtain the correct answers. A common error was to use $f(-1)$ instead of $f(1)$. Some misread the question and put both remainders equal to 7 . Many candidates found $a+b=0$ and then made a mistake and used this as $a=b$. Another common error occurred when solving the two equations by subtracting one from the other and making mistakes with the - signs. There were more errors than might be expected in the solution of the two relatively simple simultaneous equations.

## Question 5

Although this was accessible to all candidates and marks were gained by the vast majority of candidates, it was a little disappointing to see some of the errors made. The fact that BC could not be the largest side of the triangle did not stop answers of over 700 m for BC , for example; a quick check of the working might have found the error. The most common mistake in (a), however, was to evaluate $740000-700000 \cos 15^{\circ}$ as $40000 \cos 15^{\circ}$, and so $\mathrm{BC}=197 \mathrm{~m}$ ( 3 s.f.) was often seen; it was disappointing to see this error at this level.

In part (b) the most common strategy was to use the sine rule to find angle $A B C$. For the vast majority of such candidates AC was the largest side of the triangle, but there was a widespread lack of awareness that, therefore, ABC was the largest angle in the triangle. Whilst the good candidate's correctly gave the obtuse angle, the most common answers for angle ABC were $45.7^{\circ}$ or $45.8^{\circ}$, which resulted in a maximum of 2 marks being available for this part. Candidates who used the cosine rule to find angle ABC, or who found angle ACB first, were much more successful.

## Question 6

Almost all candidates correctly obtained the values -4 and 2 in part (a).
In part (b) most candidates knew they had to integrate to find the area, and most did so correctly, with only a few differentiating and only a few mistakes in the integration. They generally used the limits correctly, though a surprising number split from -4 to 0 and from 0 to 2 . It was quite common to leave the final answer as 24 . Arithmetic with fractions was invariably well executed. Those who found the rectangle area separately usually did this correctly. Many candidates subtracted the functions before integrating and this often led to the predictable errors of incorrect subtraction, or of obtaining a negative area after subtraction the wrong way round. Some did realise that the area must be positive, but the reason for the sign change was not always explained.

## Question 7

Logarithm questions tend to produce a whole spectrum of solutions, and part (a) in particular gave a good indication as to which pupils had fully grasped the rules of logarithms. Weaker candidates, however, often were able to gain marks in part (b).

In part (a) many candidates did score full marks but there was also a large number of responses which displayed little or no understanding, with such statements as $\log _{2}(5 x+4)=\log _{2} 5 x+\log _{2} 4$ and others such as $\log 2 x-\log (5 x+4)-\log 8=0$ implies that $2 x-(5 x+4)-8=0$.

A number of candidates made the common error when attempting to use the subtraction rule for logs, for example writing $\log 2 x-\log (5 x+4)=\frac{\log 2 x}{\log (5 x+4)}$, but then went on to reach $x=\frac{4}{11}$. This was covered by a special case in the mark scheme, but candidates should be reminded that a correct numerical answer may not score full marks if errors are seen in the working, as in this case.

A variety of approaches were observed with some candidates expressing $\log _{2} 2 x$ as $\log _{2} 2+\log _{2} x$ and others choosing to approach the question by first changing 3 to $\log _{2} 8$ or -3 to $\log _{2}\left(\frac{1}{8}\right)$. It was sad to see good $\log$ work followed occasionally by such a basic error as $x=\frac{11}{4}$ following $11 x=4$. Those who scored full marks usually gave an exact fractional solution as opposed to a recurring decimal.

In part (b) the power and addition rules for logs were evidently more widely known and easily applied, as most students were able to access the two method marks. Failure to understand 'express $y$ in terms of $a$ ' resulted in many students leaving their answer as $8 y=a^{5}$ or more commonly $\log _{a} 8 y=5$. Some, however, tried to rearrange to make ' $a$ ' the subject of their answer.

## Question 8

This question involved several different areas of work, area, volume, algebraic manipulation and calculus, and although a significant number of candidates produced clear and wellstructured solutions, this proved a discriminating question for many candidates.

Candidates who had learnt the formula for the volume and surface area of a cylinder usually gained at least three marks in part (a). However with a 'show that' question it is important to explain each line of the working. In far too many cases the $6 \pi r^{2}$ appeared with no reference to the cost multiplied by area. A fully correct solution also required the candidates to segregate the surface area from the cost. Far too many erroneously just replaced the 'surface area' with ' $C$ ' or 'cost' thus forfeiting the last mark. Around $5 \%$ of the candidates misread the volume as 75 instead of $75 \pi$. They lost at most one accuracy mark in (a) but many recovered in (b) and used the correct form of $C$ given in the question. A number of candidates left this section blank. The final line often appeared after several attempts and much crossing out and errors. Common errors included incorrect formulae, no obvious method, missing $\pi$ and no mention of "cost $=$ " in the final answer.

In part (b) many candidates correctly differentiated $C$ and dealt with the negative power of $x$ successfully. A large proportion also then set their expression equal to zero and found $r=\ldots$. There was some very poor algebraic manipulation, particularly the negative power, and the correct value of $r$ and $C$ were achieved by only half of the candidates. A number who found the correct value for $r$ failed to calculate the value of $C$.

If candidates successfully tackled Part (b) then they generally knew that for part (c) they should calculate $C^{\prime}$ and check that it was positive, to ascertain that $C$ was in fact minimised with their value of $r$. Having calculated $\frac{\mathrm{d} C}{\mathrm{~d} r}$ accurately, most found the correct expression for $\frac{d^{2} C}{d r^{2}}$.

## Question 9

This fairly standard turning point question saw a large number of excellent solutions, and was more accessible to weaker candidates than in some previous years, although the fractional powers caused difficulties for a significant number of candidates.

Although in most cases a correct first derivative was found in part (a), many candidates struggled to find a solution to $2 x-\frac{16}{\sqrt{x}}=0$. Some candidates spotted that $x=4$ is a solution, whilst some of those who saw how to solve the equation and achieved the stage $x^{\frac{3}{2}}=8$ still had issues, with many reaching the result of $16 \sqrt{ }$, clearly having evaluated $8^{\frac{3}{2}}$. Candidates who correctly squared their equation to give $4 x^{2}=\frac{256}{x}$, as opposed to the occasionally seen $4 x^{2}+\frac{256}{x}=0$, were often more successful in finding $x=4$. Providing the $x$-coordinate found was positive, there was a method mark available for finding $y$, but often this was not attempted or, less frequently, lost because $x$ was substituted in the expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$. Other poor attempts saw the use of a second derivative equated to zero which led to a forfeit of the final method mark for finding a $y$ value using an $x$ value resulting from this incorrect process.

In part (b) many candidates were able to correctly differentiate their first derivative, with very few using the alternative gradient method. However, there were some common sign slips with the second term. Incorrect statements were seen such as ' $x>0$ so minimum' or use of $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=0$ leading to an alternative value of $x$ which was then used to determine the nature of the turning point. Others listed all possible outcomes for the second derivative ( $>0$ so minimum, $<0$ so maximum, etc) but failed to identify whether the point $P$ was in fact a maximum or minimum.

## Question 10

This question was a good source of marks for many candidates.
In Q10(a), the missing values in the table were usually calculated correctly, however the second value was sometimes given as 3.633 rather than 3.634.

The Trapezium Rule was usually dealt with appropriately but the strip length was sometimes incorrectly used as $\frac{3}{7}$. More frequently, the final answer was not given to the required accuracy.

In Q10(b) the integration was often well answered but there were errors on the third and fourth terms (which involved negative and fractional powers). The limits of 1 and 4 were usually used correctly although candidates are advised to show clearly the substitution and evaluation of the limits to avoid losing unnecessary marks.

## Statistics for C2 Practice Paper Bronze Level B3

Mean score for students achieving grade:

| Qu | Max <br> score | Modal <br> score | Mean <br> \% | ALL | $\mathbf{A}^{*}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 6 |  | 92 | 5.54 | 5.80 | 5.83 | 5.73 | 5.58 | 5.37 | 5.24 | 4.25 |
| $\mathbf{2}$ | 5 |  | 89 | 4.47 | 5.00 | 4.85 | 4.74 | 4.40 | 4.14 | 3.86 | 2.38 |
| $\mathbf{3}$ | 4 |  | 90 | 3.60 | 3.99 | 3.92 | 3.66 | 3.64 | 3.25 | 3.18 | 2.25 |
| $\mathbf{4}$ | 6 |  | 89 | 5.35 | 5.92 | 5.86 | 5.69 | 5.38 | 4.91 | 4.52 | 3.21 |
| $\mathbf{5}$ | 7 |  | 72 | 5.04 |  | 6.09 | 5.55 | 4.90 | 4.46 | 3.78 | 2.86 |
| $\mathbf{6}$ | 9 |  | 86 | 7.74 | 8.95 | 8.58 | 8.06 | 7.60 | 6.89 | 6.32 | 3.59 |
| $\mathbf{7}$ | 7 | 7 | 63 | 4.41 | 6.85 | 6.47 | 5.70 | 4.84 | 3.85 | 2.76 | 1.08 |
| $\mathbf{8}$ | 10 | 10 | 54 | 5.37 | 9.35 | 8.64 | 6.95 | 5.41 | 3.87 | 2.59 | 0.99 |
| $\mathbf{9}$ | 9 | 9 | 60 | 5.38 | 8.72 | 8.18 | 7.05 | 5.75 | 4.38 | 3.12 | 1.30 |
| $\mathbf{1 0}$ | $\mathbf{1 2}$ | 12 | 85 | 10.14 | 11.85 | 11.46 | 10.83 | 10.14 | 9.32 | 8.34 | 5.88 |
|  |  |  | $\mathbf{7 6 . 0 5}$ | $\mathbf{5 7 . 0 4}$ | $\mathbf{6 6 . 4 3}$ | $\mathbf{6 9 . 8 8}$ | $\mathbf{6 3 . 9 6}$ | $\mathbf{5 7 . 6 4}$ | $\mathbf{5 0 . 4 4}$ | $\mathbf{4 3 . 7 1}$ | $\mathbf{2 7 . 7 9}$ |

